A

VINDICATION

OF

Sir *ISAAC NEWTON*'s

Principles of FLUXIONS,

AGAINST THE

OBJECTIONS

Contained in the

ANALYST.

By *J. WALTON*.

K

——*Si quid novisti rectius istis,*
Candidus imperti: Si non, his utere mecum. Hor.

In the Fulness of his Sufficiency he shall be in Straits: Every Hand of the WICKED *shall come upon him.* JOB.

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A
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OF
Sir *Isaac Newton*'s Principles, &c.

UNDER Pretence of ſome Abuſes committed by Mathematicians, in virtue of the Authority they derive from their Profeſſion, the Author of the *Minute Philoſopher*, in a *Libel* called the *Analyſt*, has declared them Infidels, makers of Infidels, and ſeducers of Mankind, in Matters of the higheſt Concernment: This he profeſſes to have done, not from any real Knowledge of his own, but from the credible Information of others; but he has neither produced his Informers, nor proved the Accuſation in any one Inſtance, and therefore it is Defamatory.

 But

But they aſſume an Authority, it ſeems, in Things foreign to their Profeſſion, and undertake to decide in Matters whereof their Knowledge can by no Means qualify them to be competent Judges: And as this Practice, if not prevented, may be of dangerous Conſequence; he has undertaken to enquire into the Object, Principles and Method of Demonſtration, admitted by the Mathematicians of the preſent Age, with the ſame Freedom, he ſays, they pretend to treat the Principles and Myſteries of Religion; to the End, that all Men may ſee what Right they have to lead, or what Encouragement others have to follow them.

And whereas Sir *Iſaac Newton* has preſum'd to interpoſe in *Prophecies* and *Revelations*, and to decide in religious Affairs; it has been thought proper to begin with his Method of *Fluxions*, and to try what could be done with that Method, with the Inventor himſelf, and with his Followers: And what has been done with them, every intelligent Reader is able to judge.

If this Writer may be credited, the Objects about which the Method of *Fluxions* is converſant, are difficult to conceive or imagine diſtinctly; the Notions are moſt abſtracted incomprehenſible Metaphyſics, not to be admitted for the Foundations of clear and accurate

curate Science; the Principles are obſcure, repugnant, precarious; the Arguments admitted in Proofs, are fallacious, indirect, illogical; and the Inferences and Concluſions not more juſt, than the Conceptions of the Principles are clear.

How far the Credulous and Injudicious may become infected by this uncommon way of treating Mathematicks and Mathematicians, is not eaſy to foreſee; and therefore it will be neceſſary to give a ſhort Account of the Nature of *Fluxions*, and of the Objects about which the Method is converſant; and when it ſhall be made apparent, that this Author has not underſtood the Metaphyſics he would refute, it will not be difficult to defend the Principles and their Demonſtrations, from any Imputations of Fallacy or Repugnancy, which have yet been pointed at by him, or any other Writer.

"In the Method of *Fluxions*, Sir *Iſaac* " *Newton* conſiders mathematical Quantities, " not as compoſed of the ſmalleſt Parts, but " as deſcribed or generated by continual Mo- " tion. Lines are deſcribed, and by being " deſcribed are generated, not by an Appo- " ſition of Parts, but by the Motion of " Points; Surfaces by the Motion of Lines, " Solids by the Motion of Surfaces, Angles " by the Rotation of their Sides, Times by

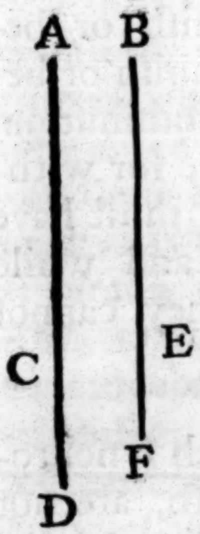
" a

" a continual Flux, and ſo of the reſt. And " by conſidering that Quantities, increaſing " in equal Times, and generated by increa- " ſing, become greater or leſs, according as " the Velocity with which they increaſe, and " are generated, is greater or leſs, he found " a Method of determining the Quantities " themſelves from the Velocities of the Mo- " tions, or of the Increments with which " they are generated; calling the Velocities " of the Motions or of the Increments *Fluxi-* " *ons*, and the Quantities generated *Fluents*.

The momentaneous Increments or Decrements of flowing Quantities, he elſewhere calls by the Name of *Moments*, and conſiders the Increments as added or affirmative Moments, and the Decrements as ſubducted or negative ones: By Moments we may underſtand the naſcent or evaneſcent Elements or Principles of finite Magnitudes, but not Particles of any determinate Size, or Increments actually generated; for all ſuch are Quantities themſelves, generated of Moments.

The Magnitudes of the momentaneous Increments or Decrements of Quantities are not regarded in the Method of Fluxions, but their firſt or laſt Proportions only; that is, the Proportions with which they begin or ceaſe to exiſt: Theſe are not their Proportions immediately before or after they begin or ceaſe to

to exiſt, but the Proportions with which they begin to exiſt, or with which they vaniſh. If the Lines A C and BE are ſuppoſed to be generated in the ſame Time, by the Motions of the Points A and B, to C and E; and if by continuing the Motions of thoſe Points to D and F, they generate DC and EF, ſynchronal Increments of AC and BE; it is evident that the Points D and F may flow



back in the ſame Time to C and E, and by flowing back perpetually, leſſen the Magnitudes of thoſe Increments 'till at laſt they vaniſh together, when the Points D and F come to coincide with C and E: Now the ultimate Ratio of thoſe Increments is that Ratio with which they vaniſh and become nothing; or the Ratio with which they ceaſe to be: And the firſt Ratio of them is the Ratio with which they begin to exiſt, at the very firſt ſetting out of the Points from C and E towards D and F.

Hence, if the deſcribing Points move back to C and E, in the ſame Time wherein by moving forward they generated the Increments DC and EF; and in returning have every where the ſame Velocities, at certain Diſtances from C and E, which they had at thoſe

those Distances in going forward; the last and first Ratios of the Increments will be equal, or they will vanish, and become nothing, with the very same Ratio with which they began to exist.

Hence likewise it appears, that to obtain the last Ratio of synchronal Increments, the Magnitudes of those Increments must be infinitely diminish'd. For their last Ratio is the Ratio with which they vanish or become nothing: But they cannot vanish or become nothing, by a constant Diminution, 'till they are infinitely diminish'd; for without an infinite Diminution they must have finite or assignable Magnitudes, and while they have finite Magnitudes they cannot vanish.

The ultimate Ratios with which synchronal Increments of Quantities vanish, are not the Ratios of finite Increments, but Limits which the Ratios of those Increments attain, by having their Magnitudes infinitely diminish'd: The Proportions of Quantities which grow less and less by Motion, and at last cease to be, will, in most Cases, continually change, and become different in every successive Diminution of the Quantities themselves: And there are certain determinate Limits to which all such Proportions perpetually tend, and approach nearer than by any assignable Diffe-

Difference, but never attain before the Quantities themselves are infinitely diminish'd; or 'till the Instant they evanesce and become nothing. These Limits are the last Ratios with which such Quantities or their Increments vanish or cease to exist; and they are the first Ratios with which Quantities or the Increments of Quantities, begin to arise or come into Being.

Quantities, and the Ratios of Quantities, which constantly tend to an Equality, by a Diminution of their Difference, and before the End of some finite Time approach nearer to an Equality than by any assignable Difference, at last become equal. For they become equal when the Difference between them vanishes or becomes nothing; and it will vanish or become nothing by being infinitely diminished, If the Quantities A C and A D perpetually tend to an Equality, either by the Motion of the Point D to C, or by that of C to D, they will become equal, and their Ratio a Ratio of Equality, when their Difference C D, by a constant Diminution, vanishes and becomes nothing, which it will do under a Coincidence of the two Points in C or D; and then either A D becomes A C, and so $\frac{AD}{AC}$ or $\frac{AC}{AC}$ is a Ratio of Equality, or else A C becomes A D and $\frac{AD}{AC}$ becomes $\frac{AD}{AD}$; which is also a Ratio of Equality.

The Fluxions of Quantities are very nearly as the Increments of their Fluents generated in the least equal Particles of Time. If CD and EF be Increments of the Fluents AC and BE, deſcribed in the leaſt equal Particles of Time; the Fluxions in the Points C and E will be nearly as the Increments DC and EF. For from the exceeding Smallneſs of the Times, it is evident that the Points D and F, muſt be extreamly near to C and E; and by Conſequence, however the Velocities are accelerated or retarded through the Spaces CD and EF, they will be very nearly the ſame in D and F, as they were in C and E: But Velocities which are very nearly uniform, will be very nearly proportional to the Spaces deſcribed by them in equal Times; and therefore the Velocities in the Points C and E, which are the Fluxions of AC and BE in thoſe Points, will be very nearly as the Increments DC and EF, deſcribed in the leaſt equal Particles of Time.

The Fluxions of Quantities are accurately in the firſt or laſt Proportions of their naſcent or evaneſcent Increments. Thus the Fluxions of AC and BE, in the Points C and E, are in the firſt or laſt Ratio of the Increments CD and EF. For the firſt or laſt Ratio of the Increments CD and EF, is the Ratio with which they begin or ceaſe to exiſt: But the Ratio with which they begin or ceaſe to ex-

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iſt, is the ſame with the Ratio of the Velocities in C and E, which are the Fluxions in thoſe Points; and conſequently the Fluxions in C and E are in the firſt or laſt Ratio of the Increments C D and E F.

The Fluxions of Quantities are only the Velocities with which thoſe Quantities begin to be generated or increaſed; or the Velocities with which the generating Quantities begin to ſet out; not the Velocities they have after moving through Spaces of any finite or aſſignable Magnitudes: And therefore if two mathematical Quantities ſet out together, and begin to move with Velocities which are as *a* and *b*, they muſt begin to deſcribe Spaces in the ſame Proportion with *a* and *b*; or the Proportion withwhich theSpaces begin to exiſt, or to be deſcribed, muſt be the ſame with that which the Velocities have at the very Beginning of the Motion. For in the very Beginning of the Motion there is neither any Change of Velocity from Acceleration or Retardation, nor Difference of Time.

Hence it appears, that to obtain the Ratios of Fluxions, the correſponding ſynchronal or iſochronal Increments muſt be leſſened *in infinitum*. For the Magnitudes of ſynchronal or iſochronal Increments muſt be infinitely diminiſhed and become evaneſcent, in order to obtain their firſt or laſt Ratios, to

which Ratios the Ratios of their correſponding Fluxions are equal.

Hence likewiſe it appears that the Moments of like Quantities, compared with each other, are in Ratios compounded of the Ratios of the generating Quantities, taken when firſt they begin to move, and of the Velocities with which they ſet out; or in Ratios compounded of the Ratios of the generating Quantities when firſt they begin to move, and of the firſt Ratios of the Spaces deſcribed by them in equal Times. The Moments of Lines, therefore, are as the generating Points, and as the Velocities with which they begin to move, taken together: The Moments of Surfaces, which become greater or leſs by carrying of moveable Lines along immoveable ones, are in Ratios compounded of the moving Lines, and of their firſt Velocities, or firſt *Ratios of the Increments* which begin to riſe with thoſe Velocities: And the whole Motion by which Squares or Rectangles begin to alter, either from an Augmentation or Diminution of their Sides, is the *Sum of the naſcent Motions* of thoſe Sides, or the Sum of the naſcent Increments ariſing with the *firſt Motions* of the Sides: For the Proportion of naſcent Increments is the ſame with that of the Motions with which they begin to be generated.

From

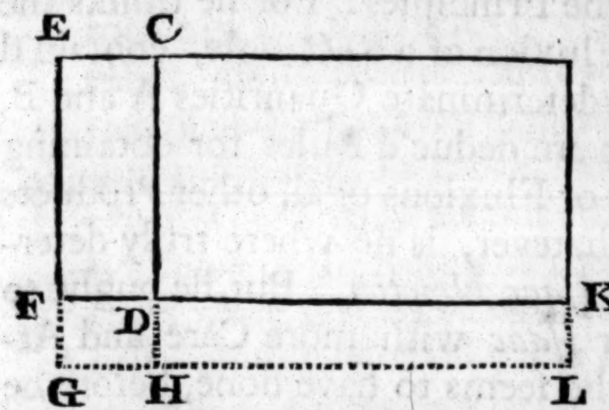
From this ſhort Account of the Nature of Fluxions, compared with the *Analyſt*, it appears that the Author of that Paper is greatly miſtaken in the Object of 'em; and he is alſo miſtaken in the Principles: For he thinks the Moment or Fluxion of a Rectangle, contain'd under two indeterminate Quantities A and B, from whence are deduc'd Rules for obtaining the Moments or Fluxions of all other Products or Powers whatever, is no where truly determin'd by Sir *Iſaac Newton*. But he ought to have read Sir *Iſaac* with more Care and Attention than he ſeems to have done, before he ſet up to decide and dictate in Matters of this Nature; and he wou'd do well yet to read him with Attention.

If any Rectangle CK be increaſed from an Augmentation of its Sides by Motion, ſo as that DK becomes LG in the ſame Time that DC becomes EG; the Moment of that Rectangle is the Sum of the Rectangles of DK into the Moment of DC, and of DC into the Moment of DK: That is, putting A and B for the Sides DK and DC, and a and b for their reſpective Moments, the Moments of the Rectangle AB will be $Ab+Ba$.

For the Gnomon CGK in the Inſtant it begins or ceaſes to exiſt, is the Moment of the Rectangle CK: But the firſt or laſt Ratio of that Gnomon to the Sum of the Rectangles

LD

LD and FC is a Ratio of Equality: For the Difference between the Gnomon and the Sum of those Rectangles perpetually lessens by a constant Diminution of the Increments FD and DH, or by an Approach of the Points F and H towards D; as will be manifest on taking the



Ratio between the said Gnomon and the Sum of the Rectangles, at several Distances of the Points F and H from D: For whatever be the Magnitudes of *a* and *b*, when F and H first begin to move back towards D, the Gnomon CGK and the Sum of the Rectangles LD and FC, will be as $Ab+Ba+ba$ and $Ab+Ba$; when those Points, by moving towards D, have lessen'd the Increments of DK and DC to $\frac{1}{2}a$ and $\frac{1}{2}b$, the Gnomon and the Sum of the Rectangles will be as $Ab+Ba+\frac{1}{2}ba$ and $Ab+Ba$; when they have lessen'd the Increments to $\frac{1}{4}a$ and $\frac{1}{4}b$, the Gnomon and the Sum of the Rectangles will be as $Ab+Ba+\frac{1}{4}ab$ and $Ab+Ba$; and as $Ab+Ba+\frac{1}{8}ab$ and $Ab+Ba$, when they have lessen'd those Increments to $\frac{1}{8}a$ and $\frac{1}{8}b$. Hence it appears, that under a constant Diminution of the Increments *a* and *b*, by the Motion of the Points F and

F and H towards D, the Gnomon CGK and the Sum of the Rectangles CF and DL, constantly tend to an Equality by a continual Diminution of their Difference FH; and that they become equal, and their Ratio becomes a Ratio of Equality, in the Instant that Difference vanishes and the Points F and H coincide with D; or, in other Words, the Gnomon and the Sum of the Rectangles LD and FC begin or cease to be under a Ratio of Equality: And therefore the Sum of those Rectangles, or $Ab+Ba$, is the Moment of AB.

Hence, the Gnomon CGK, or $Ab+Ba+ab$, found by taking the Difference between the Rectangles EL and CK, or by deducting the Rectangle AB from a Rectangle contain'd under the Sides A and B increased by their whole Increments, is not the Moment or Fluxion of the Rectangle AB, except in the very Instant when it begins or ceases to exist: And this will also appear by considering it in another Light. For the Moment of the Rectangle CK, or the Motion with which it first begins to alter, either by increasing or decreasing, is the Sum of the nascent Motions of its Sides; and the nascent Motions of its Sides, are measur'd by their respective Magnitudes in the very Instant they first begin to change, and by the Velocities with which they begin to move taken together; and the Velocities with which the Sides begin

gin to move being in the first Ratio of the momentaneous Spaces which arise with 'em; it follows, that the Sum of the nascent Motions of the Sides, is the Sum of DK multiply'd into DH in its nascent State, and of CD multiply'd into DF in its nascent State: But DH and DF in their nascent States, are the Moments of DC and DK; and therefore the whole Moment of the Rectangle AB, is $Ab+Ba$.

In determining the Moments of Quantities, Sir *Isaac Newton* expresly tells us, that we are only to consider the Ratios with which they begin or cease to exist; and to obtain those Ratios, it is not necessary that the isochronal Increments shou'd have finite Magnitudes. " *Cave tamen intellexeris particulas* " *finitas*, says he, *Particulæ finitæ non sunt* " *Momenta, sed Quantitates ipsæ ex Momentis* " *genitæ. Intelligenda sunt Principia jamjam* " *nascentia finitarum Magnitudinum. Neque* " *enim spectatur in hoc Lemmate magnitudo* " *Momentorum, sed prima nascentium propor-* " *tio.* And in another Place, " *Fluxiones* " *sunt quam proximè ut Fluentium Augmenta* " *æqualibus Temporis particulis quam minimis* " *genita, &, ut accurate loquar, sunt in pri-* " *mâ ratione Augmentorum nascentium; exponi* " *autem possunt per lineas quascunque, quæ sunt* " *ipsis proportionales.* And again, *Siquando* " *facili rerum conceptui consulens dixero Quan-*

" *titates*

" *titates quam minimas, vel evaneſcentes, vel*
" *ultimas; cave intelligas quantitates magni-*
" *tudine determinatas, ſed cogita ſemper dimi-*
" *nuendas ſine limite.*

From theſe Paſſages it appears, that the Gnomon CGK in its naſcent or evaneſcent State only, or in the Inſtant it begins or ceaſes to exiſt, is the Moment or Fluxion of the Rectangle CK; and in a naſcent or evaneſcent State, when only the Increments of Quantities become their Moments, its Ratio to A*b*+B*a*, which is the Sum of the Rectangles LD and FC, is a Ratio of Equality. By diminiſhing the Magnitudes of *a* and *b*, which are Increments of DK and DC, it is obvious that the Gnomon CGK diminiſhes faſter in Proportion, than the Sum of the Rectangles FC and DL does; and by diminiſhing faſter, it continually approaches to an Equality with that Sum; and attains the Equality only, when their Difference FH becomes evaneſcent, that is, when the Points F and H come to coincide with D: So that here is no Artifice or falſe Reaſoning uſed, to get rid of HF, or *ab*, that Term having no Exiſtence at the very Beginning of the Motion, or in the naſcent State of the Augments.

After Sir *Iſaac* had ſo expreſly told us what he meant by Moments and Fluxions, and by

nascent or evanescent Quantities, one wou'd imagine it impossible to have mistaken and misrepresented him in the Manner this Author has done. He seems indeed to have been led, or rather to have been deceived, by an Opinion that there can be no first or last Ratios of mathematical Quantities, or of their isochronal Increments, generated or destroy'd by Motion; imagining that no such Quantities, by any Division or Diminution whatever, can be exhausted or reduc'd to nothing: But if Lines, Surfaces and Solids, can be generated or augmented by the Motion of Points, Lines, and Surfaces, they may likewise be destroy'd or diminish'd by the Motion of the same Points, Lines and Surfaces, in returning to the Places from whence they first set out. While a generating Quantity moves back thro' the same Space it before described in moving forward, the Quantity generated, or its Augment, continually lessens; and by persevering in a State of decreasing, it must in some finite Time vanish and become nothing; and therefore mathematical Quantities, by a constant Diminution, may be reduc'd to nothing: And such as are thus generated or destroy'd in equal Times by Motion, or which arise and vanish together, will arise or vanish under certain Ratios, which are their first or last Ratios; or the Ratios with which they begin or cease to be. But it may be necessary

ceſſary to purſue this Caſe a little farther, and ſee whether Sir *Iſaac Newton*'s Demonſtration of it cannot be defended, and proved to be geometrical.

" Suppoſe any Rectangle AB augmented
" by continual Motion; and the momenta-
" neous Increments of its Sides A and B to be
" denoted by *a* and *b*; the Moment of the ge-
" nerated Rectangle will be meaſured by
" $Ab+Ba$.

" For when the Sides A and B wanted
" half of their Moment, the Rectangle was
" $\overline{A-\frac{1}{2}a}\times\overline{B-\frac{1}{2}b}$ or $AB-\frac{1}{2}Ab-\frac{1}{2}Ba+\frac{1}{4}ab$:
" And as ſoon as the Sides A and B are aug-
" mented by the other halves of their Mo-
" ments, it becomes $\overline{A+\frac{1}{2}a}\times\overline{B+\frac{1}{2}b}$, or
" $AB+\frac{1}{2}Ab+\frac{1}{2}Ba+\frac{1}{4}ab$: From this Re-
" ctangle deduct the former, and there will
" remain $Ab+Ba$: Therefore the Incre-
" ment of the Rectangle AB, generated with
" *a* and *b*, the whole Increments of the Sides,
" is $Ab+Ba$.

Now, in determining the Moment of a Rectangle, there is nothing to be conſidered when it firſt begins to be augmented by the Motions of its Sides, but the Sides themſelves, and the Velocities with which they begin to move; or the Sides and the firſt Ratio of the

Spaces described by them. And therefore the true Moment of the Rectangle AB, or the Law according to which it begins to be augmented, on the Principles of Sir *Isaac Newton*, will only be the Sum of the Rectangles Ab and Ba; for the Sides A and B begin to move with Velocities which are as b and a: But this Moment Ab+Ba, is manifestly equal to the Difference between the Rectangles $\overline{A+\frac{1}{2}a} \times \overline{B+\frac{1}{2}b}$ and $\overline{A-\frac{1}{2}a} \times \overline{B-\frac{1}{2}b}$; and therefore Sir *Isaac*'s Determination of it is geometrical.

From the foregoing Principle so demonstrated, the general Rule for finding the Moment or Fluxion of any Power of a flowing Quantity, is easily deduc'd: It is easy, from hence, to infer that the Moment or Fluxion of A^n is as nA^{n-1}, or that the Fluxion of x^n is as nx^{n-1}. But because this is also determined in a manner seemingly different, by Sir *Isaac*, in his Introduction to the *Quadrature of Curves*, the Author of the *Analyst* observes, " That there seems to have been some " inward Scruple or Consciousness of Defect " in the foregoing Demonstration." And he repeats the same Reflection in another Place; adding withal, " That Sir *Isaac* was not e- " nough pleased with any one Notion steddi- " ly to adhere to it. But Reflections of this Nature deserve no Regard unless it be allowable, by way of Return, to observe that the

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Perſon who makes 'em has very often been guilty of like Practices himſelf.*

The Proof given in the Introduction to the *Quadratures*, is ſaid to be a moſt inconſiſtent way of arguing; as proceeding to a certain Point of the Demonſtration upon Suppoſition of an Increment, and then in a fallacious Manner, ſhifting the Suppoſition to that of no Increment; and to ſhew the Inconſiſtency with greater Force, a Lemma is premiſed by Way of Axiom; as if ſome very obvious and natural Application of an apparent Truth, wou'd at once overturn the Whole of Sir *Iſaac*'s Demonſtration: But that Lemma, however true in it ſelf, is no Way pertinent to the Caſe for which it was intended; and therefore ſuch Inferences as are made in virtue of it, with relation to the Point in diſpute, are illegitimate, and inconſiſtent with the Rules of true Reaſoning.

Nothing is more plain and obvious, than that Quantities which begin to exiſt together under certain Proportions, and with certain Velocities, may become evaneſcent and ceaſe to exiſt, under the ſame Proportions and with the ſame Velocities; and this is all Sir *Iſaac* ſuppoſes in the Determination of the Fluxion of

* See his new *Theory of Viſion*; his *Treatiſe on the Principles of Human Knowledge*; and ſome later Undertakings of equal Importance.

of x^n; and it is not very obvious that the Lemma which this Author has hit upon, is applicable to Cases of such a Nature.

That the Reader may see how strictly Sir *Isaac Newton* has kept to the same Principle in this Determination, how steddily he adheres to the same Method, and how ill the Author of the *Analyst* has proved his Imputations; it will be necessary to pursue this Point, and consider the Proof it self.

Let it be required to find the Fluxion of x^n, supposing x to increase uniformly.

Suppose x in any finite Particle of Time to become greater than before, by a finite Increment, whose Magnitude is express'd by o. Then, in the same Time that x, by flowing becomes $x+o$, the n Power of x will become $x^n + nox^{n-1} + \frac{n^2-n}{2} o^2 x^{n-2} +$ &c. Consequently, the* Magnitudes of the synchronal Increments of x and of x^n, are to each other as 1 and $nx^{n-1} + \frac{n^2-n}{2} ox^{n-2} +$ &c. Now, let the Increments decrease by flowing back, in like Manner as they increas'd before by flowing forward, and continually grow less and less till they vanish; and their ultimate Ratio, that is, the Ratio with which they become evanescent, will be express'd by 1 and nx^{n-1}: But

* ~~i.e.~~ o ~~and~~ $nox^{n-1} + \frac{n^2-n}{2} o^2 x^{n-2} +$ &c.

But the Fluxions of Quantities are in the laſt Ratio of their evaneſcent Augments; and by Conſequence the Fluxion of x is to that of x^n, as 1 to nx^{n-1}.

In this Computation, Sir *Iſaac* endeavours to collect the Proportion with which the iſochronal Increments of x and of x^n, begin or ceaſe to exiſt: Their Proportion obtain'd on Suppoſition that o is ſomething, is allowed to be the ſame with that of 1 and nx^{n-1} $+\frac{n^2-n}{2}ox^{n-2}+$ &c. And it muſt be acknowledg'd that this Ratio has a Limit dependent on the Magnitude of o, which Limit it cannot attain before the Increments are infinitely diminiſh'd and become evaneſcent; and when, by an infinite Diminution, they become evaneſcent, no other Terms of their Ratio will be affected, ſo as to vaniſh with 'em, but ſuch as are govern'd or regulated by them: In the Inſtant therefore that o vaniſhes, $\frac{n^2-n}{2}ox^{n-2}$ and all enſuing Terms of the Series abſolutely vaniſh together; but the Terms 1 and nx^{n-1} remain invariable under all poſſible Changes of the Increments, from any finite Degrees of Magnitude whatever, even till they become evaneſcent: They therefore expreſs the laſt Ratio, under which the iſochronal Increments of x and x^n vaniſh, or the Proportion of the Velocities with which thoſe Increments ceaſe to exiſt: Sir *Iſaac Newton* then rightly retain'd

retain'd 'em for the Meaſures of the Ratio of the Fluxions of x and x^n, tho' got in virtue of his firſt Suppoſition; and the Falacy, the Inconſiſtency, lies on the Side of this Author; who wou'd have them rejected on the Authority of a Lemma not to the Purpoſe.

To make this Point ſtill more plain and obvious, I ſhall propoſe the Reaſoning in a ſtronger Light: It amounts therefore to this, or may in other Words, be thus expreſſed: If x be ſuppos'd to flow uniformly, the Fluxions of x and x^n, will be as 1 and nx^{n-1}. For in the ſame Time that x by flowing, becomes $x+o$, x^n will become $\overline{x+o}|^n$, which by the Method of infinite Series, is equal to $x^n+nox^{n-1}+\frac{n^2-n}{2}o^2x^{n-2}+$ &c. Conſequently the Increments of x and x^n, generated in the ſame Time, are o and $nox^{n-1}+\frac{n^2-n}{2}o^2x^{n-2}+$ &c. But the naſcent or evaneſcent Increment of x^n, is as its Fluxion; and in either of theſe States the Ratio of $nox^{n-1}+\frac{n^2-n}{2}o^2x^{n-2}+$ &c. to nox^{n-1} is a Ratio of Equality: For as the Magnitude of o becomes leſs and leſs, the Quantities $nox^{n-1}+\frac{n^2-n}{2}o^2x^{n-n}+$ &c. and nox^{n-1} conſtantly tend to an Equality, by a continual Diminution of their Difference;

and

and they become equal, and their Ratio becomes a Ratio of Equality; when their Difference vaniſhes; that is, in the Inſtant o becomes evaneſcent, or in the Inſtant that the Increment of x^n firſt begins to exiſt: For as they vaniſh together under a Ratio of Equality, ſo they begin to exiſt together under the ſame Ratio; and therefore in the naſcent or evaneſcent State of o, the Fluxions of x and x^n, are as o and nox^{n-1}, which are manifeſtly to each other as 1 and nx^{n-1}.

Hence it appears, that this Method of finding the Fluxion of x^n, upon a Suppoſition that x flows uniformly, is the very ſame with that of finding the Fluxion of a Rectangle, as it is deſcribed in the ſecond Book of the *Mathematical Principles:* For, as ab the Difference between $Ab + Ba + ab$ and $Ab + Ba$ grows leſs and leſs perpetually, by diminiſhing the ſynchronal Increments of the Sides of the Rectangle, and at laſt evaneſces, and in the Inſtant of its Evaneſcence, the Gnomon $Ab + Ba + ab$ becomes equal to the Sum of the Rectangles Ab and Ba; ſo $\frac{n^2-n}{2}o^2x^{n-2} + $ &c. the Difference between $nox^{n-1} + \frac{n^2-n}{2}o^2x^{n-2} + $ &c. and nox^{n-1}

D grows

grows lefs and lefs perpetually, by diminifhing the Increment o, and at laft evanefces, and in the Inftant of its Evanefcence $nox^{n-1} + \frac{n^2-n}{2} o^2 x^{n-2} +$ &c. becomes equal to nox^{n-1}: And as the Gnomon $Ab + Ba + ab$ is not the Moment or Fluxion of the Rectangle AB, but in the Inftant of its becoming equal to $Ab + Ba$, fo $nox^{n-1} + \frac{n^2-n}{2} o^2 x^{n-2} +$ &c. is not the Moment or Fluxion of x^n, but in the Inftant of its becoming equal to nox^{n-1}.

The Author of the *Analyft*, therefore, is greatly miftaken, in thinking Sir *Ifaac* found the Fluxion of x^n, by a Method different from that he ufed in finding the Fluxion of a Rectangle, contain'd under two flowing Quantities: He fteadily adheres to one and the fame Method; namely, that of taking the firft or laft Ratios of Quantities, or of their ifochronal Increments, for the Meafures of the Ratios of their Fluxions; and ufes no illegitimate Artifice to obtain thefe firft or laft Ratios; unlefs it be accounted illegitimate to fuppofe that mathematical Quantities can be generated and deftroyed by Motion.

It

It is pretended, " That the Method for
" finding the Fluxion of a Rectangle of two
" flowing Quantities, as it is ſet forth in the
" *Treatiſe of Quadratures*, differs from that
" found in the *ſecond Book of the Principles*,
" and is in Effect the ſame with that uſed in
" the *Calculus differentialis:* For the ſuppo-
" ſing a Quantity infinitely diminiſh'd, and
" therefore rejecting it, is in Effect the re-
" jecting an Infiniteſimal." But if this Author deduces the Rule from the firſt Propoſition in the *Treatiſe of Quadratures*, and conſiders it ever ſo little, he will find it the very ſame with that ſet down in the *ſecond Book of the Principles:* And it is doubtleſs in Effect too the ſame with that uſed in the *differential Calculus*, ſo far as different Methods *can* effect the ſame Thing, but no farther: For Quantities are not rejected in the Method of Fluxions, as in the *differential Calculus*, on Account of their exceeding Smallneſs.

" But according to the received Principles
" it is evident, ſays he, that no geometrical
" Quantity, by being infinitely diminiſh'd,
" can ever be exhauſted or become nothing."
Now, on the received Principles of Fluxions, this is a direct Abſurdity. For theſe Principles ſuppoſe that mathematical Quantities

 can

can be generated by Motion, which he has not yet thought proper to contradict; and consequently they may also by Motion be destroy'd: For Quantities, and the Augments of Quantities, which in some finite Time are produc'd by Motion, may perpetually grow less and less by reverting that Motion; and by constantly growing less and less, they may come to be infinitely diminished, or to be less than any assignable Qnantities, and from being less than any assignable Quantities, the Motion still persevering, theymust at last vanish and become nothing; otherwise it might be contended, that a Body setting out from any Place, and in any finite Time, describing a certain Length, could never, by moving back and returning in the same Line, arrive at the Place from whence it first set out.

Upon the whole then it appears, that the Method of Fluxions, as described by Sir *Isaac Newton*, in his Introduction to the *Quadrature of Curves*, and in the *second Book of his Mathematical Principles*, is not that wretched unscientifical Knack set forth in the *Analyst*; but a Method founded upon obvious, rational, accurate and demonstrative Principles. It likewise appears, that the Conclusions do not arise from illegitimate tentative Ways or Inductions; but follow from such Premises, and by such Arguments, as are most confor-

mable to the Rules of Logic and right Reaſon: All the Skill and Dexterity therefore by this Author ſhewn, in the Inveſtigation of contrary Errors correcting each other, are vain and impertinent. He has miſtaken the Doctrine of Fluxions; and by not rightly diſtinguiſhing its Principles from thoſe of the *differential Calculus*, has impoſed a falſe Meaſure of Moments upon his Readers; and arguing from that falſe Meaſure, has unjuſtly charg'd Sir *Iſaac* with Errors ariſing from it: And, to mend the Matter, has inſtituted Computations, to ſhew how thoſe Errors redreſs one another, and how Mathematicians by Means of Errors bring forth Truth, and not Science.

The Diſpute between the Followers of Sir *Iſaac Newton*, and the Author of the *Analyſt*, is not about the Principles of the *differential Calculus*, but about thoſe of Fluxions; and it is whether theſe Principles in themſelves are clear or obſcure, and whether the Inferences from them are juſt or unjuſt, true or falſe, ſcientific or otherwiſe: We are not concerned about Infiniteſimals or minute Differences, but about the Ratios with which mathematical Quantities begin or ceaſe to exiſt by Motion; and to conſider the firſt or laſt Proportions of Quantities, does not imply that ſuch Quantities have any finite Magnitudes:

nitudes: They are not the Proportions of firſt or laſt Quantities, but Limits of Ratios; which Limits the Ratios of Quantities attain *only* by an infinite Diminution of their Magnitudes, by which infinite Diminution of their Magnitudes they become evaneſcent and ceaſe to exiſt. If therefore Quantities may ceaſe to exiſt by Motion, and if the Ratios with which they become evaneſcent be truly determin'd, it will follow that there are no Errors, however ſmall, admitted in the Principles of Fluxions; and if no Errors be admitted in the Principles, there can be none in the Concluſions, nor any to be accounted for in the Arguments by which thoſe Concluſions are deduc'd from their Premiſes. The Hints therefore, which this Author has condeſcended to give the Mathematicians for aſcertaining the Truth of their Concluſions, by means of contrary Errors deſtroying each other, will probably be left to be further extended and apply'd by himſelf, to all the *good Purpoſes* he pleaſes to extend and apply them; as having more Leiſure, and a Science more tranſcendental *, and perhaps a much greater Curioſity for ſuch Matters, than they have.

* A *Philoſophia prima*, a certain tranſcendental Science ſuperior to and more extenſive than Mathematics, which, he ſays, it might behove our modern *Analyſts* rather to learn than deſpiſe.

It

It has been obſerv'd before, that Fluxions may be expounded by any Lines which are proportional to them; and ſo the Analyſis may be inſtituted, by conſidering the mutual Relations or Proportions of Fluxions themſelves. To this it is objected, "That if, in "order to arrive at theſe finite Lines pro-"portional to Fluxions, there be certain "Steps made uſe of which are obſcure "and inconceivable, it muſt be ac-"knowledged, that the Proceeding is not "clear, nor the Method ſcientific." But there may be many Steps obſcure and inconceivable to Perſons who are unacquainted with Sir *Iſaac Newton*'s Method of firſt and laſt Ratios, with his Doctrine of Fluxions, and with his Principles of Motion; and yet thoſe Steps may appear very different to others who have duly conſider'd them: And therefore, 'till it be made apparent from geometrical Principles, that the fluxional Triangle, which evaneſces upon the returning of the Ordinate of any Curve to the Place from whence it firſt ſet out, cannot in its laſt Form, that is, in the Form that it has at the Inſtant it becomes evaneſcent, be ſimilar to a Triangle contain'd between the Tangent, the Abſciſs extended and the Ordinate of the ſame Curve; or 'till it be proved that no Tri-

angle, which is capable of becoming evaneſcent by a Diminution of its Sides from Motion, can be ſimilar in its laſt Form to any plain Triangle whatſoever; we ſhall continue to expound Fluxions by ſuch Right Lines as are proportional to them; and do aſſert, that the Proceeding is clear, and the Method ſcientific.

F I N I S.